Struve Geodetic Arc
2006 International Conference
»The Struve Arc and Extensions in Space and Time»
Haparanda and Pajala, Sweden, 13-15 August 2006

F. W. BESSEL AND GEODESY

T. Viik

Tartu Observatory, Tartumaa 61602, ESTONIA
Tartu University, Ülikooli str. 18, 50090, ESTONIA

1. Introduction

Renowned mathematician, astronomer and geodesist Friedrich Wilhelm Bessel started his geodetical research when it was already clear that Isaac Newton had been right in theorizing that the Earth was oblate. This had been ascertained by the geodetical measurements of meridian arc between Tornio and Kittisvaara by Pierre Louis Moreau de Maupertuis and Alexis Claude Clairaut in 1736 - 1737 [1], remeasured by Jöns Svanberg in 1801-1803 [2], and by Charles-Marie La Condamine and Pierre Bouguer in 1735 - 1743 between Tarqui and Cotchesqui in Peru [3]. At the same time all the attempts to express the figure of the Earth as a perfect ellipsoid failed. This called for an explanation that the curvature of the meridians along the latitude was not constant, consequently the Earth was not a perfect ellipsoid. By and by the scientists were convinced that the figure of the Earth cannot be represented by simple mathematical formulae at all.

Still the scientists wanted to specify the parameters of the ellipsoidal approximation and also to learn how much the figure of the Earth differed from that of an ellipsoid.

This goal was to be accomplished by meridian arc measurements and by determining the gravitational force at different points of the Earth. At that time the first goal could be achieved only by triangulation and for the other goal the scientists measured the gravitational force as a function of latitude. Though Bessel dedicated a lot of his energy and time to both of these goals, only the first of them will be considered in this paper. All the Bessel's papers on geodesy were collected in the third volume of the book edited by R. Engelmann [4]. The majority of these papers was previously published in

Astronomische Nachrichten [5 - 12].

2. Theory of geodetic measurements in Bessel's works

Bessel's first study in geodesy was connected with the Danish arc measurement by Heinrich Christian Schumacher in Holstein which Schumacher directed from 1817. These studies were in the form of letters to Schumacher who later published them in his journal Astronomische Nachrichten. Bessel studied the problem of substituting the triangles on an ellipsoid by triangles on a sphere. If this was possible in the framework of given accuracy then one could easily substitute the spherical triangle by a planar one taking into account that all angles of that triangle should be decreased by one third of the spherical excess [5].

The transition from measuring triangles on the ellipsoid's surface to measuring spherical triangles is complicated also by the fact that on an ellipsoid the triangles are formed by geodesics, i.e. by the shortest possible lines between two points on that surface. But the measurements by using levels give us the angles between the vertical sections of the Earth. Bessel was the first to indicate that these triangles do not coincide and elaborated a method how to convert one to another. It appeared that this conversion was possible only for triangles with sides less than 150 km if one wanted to maintain a passable accuracy.

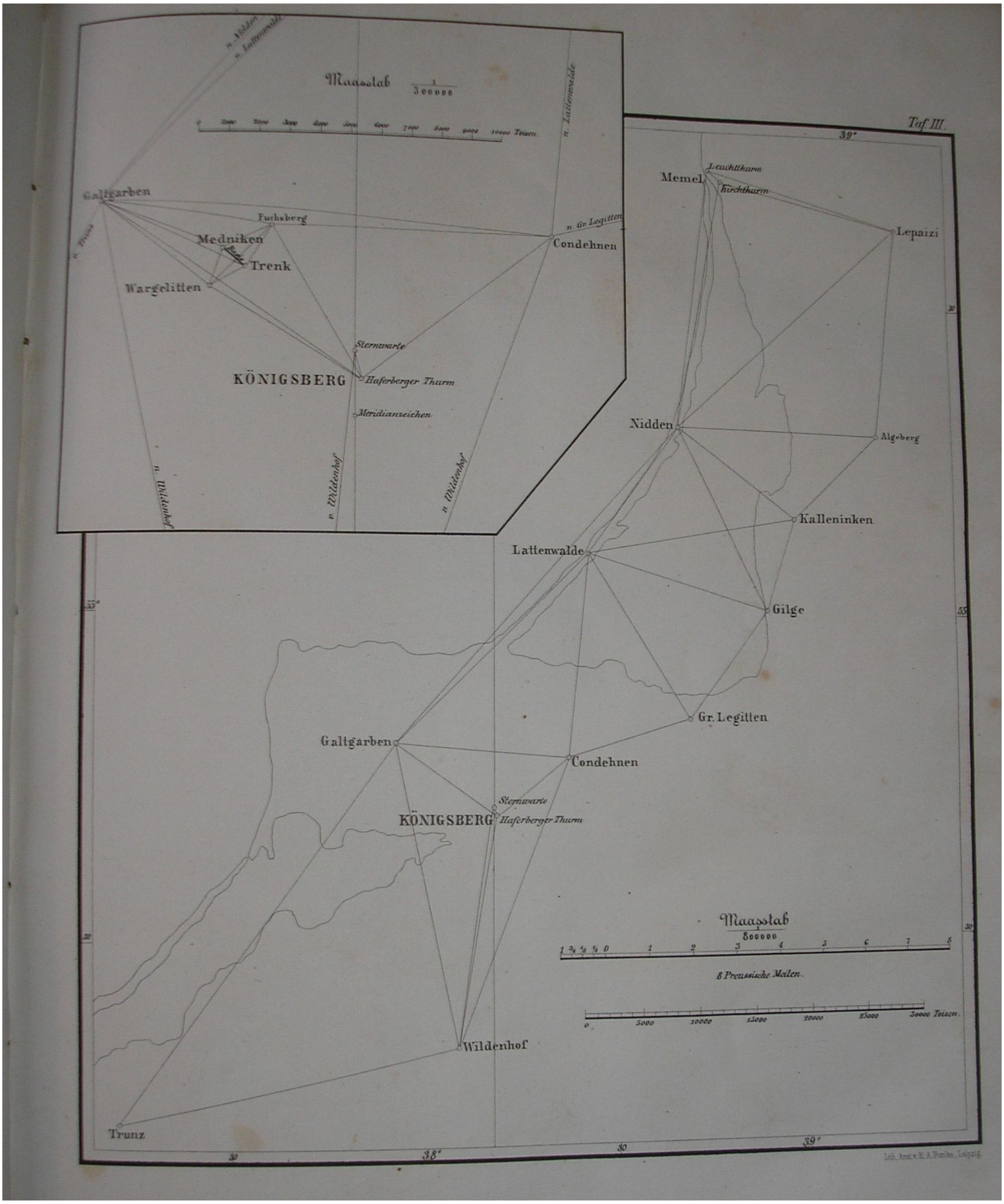
In 1826 Bessel solved one of the basic problems in geodesy – how to find the latitude and the difference in longitudes of a point *B* by measuring the latitude at point *A* and the angle between the meridian and the direction to point *B* [6] . Gauss had solved this problem by using his theory of surfaces but this solution was accurate enough only for rather small distances. Bessel gave his classical solution in trigonometric series and this solution was very general. Widely used, too, since Bessel compiled also large tables which were usable for any ellipsoidal model of the Earth.

Bessel gave also the definition of *geoid*, though the term itself was coined by a German mathematician Johann Benedict Listing only in 1873. In a paper published in 1837 Bessel wrote: *"Es ist ein wesentlicher Unterschied zwischen der physischen und der geometrischen Figur der Erde.... Denkt man sich also die Erde mit einem Netze von Kanälen überzogen, welche mit dem Meere in Verbindung sind und durch dieses gefüllt werden, so fällt die Oberfläche des ruhigen Wassers in denselben mit der geometrischen Oberfläche der Erde zusammen"* [8]. Nowadays the surface of a geoid is described exactly in this way.

Bessel studied also the so-called general reductional problem in geodesy. He investigated the influence of the difference between the geoid and the ellipsoid on the results of the astronomo-geodetical measurements. He showed that however small the differences one should take these into account. He proposed to consider as basic points not the points themselves but the points of their normal projection on the surface of ellipsoid.

3. Geodetic measurements in practice

Bessel's most important contribution to practical geodesy was the arc measurement in Prussia. The motive for this measurement was a plan to join the Russian and the West-European triangulation chains at their closest points near Königsberg where Bessel



worked as the university Professor of astronomy and mathematics. It was in 1829 when the Russian government wrote a letter to the Prussian government in which they expressed a wish that director of Königsberg University Observatory Professor Bessel should join these two chains. The triangulation in the southern parts of Russia was then ongoing and lead by major-general Carl Tenner. Bessel was quick to understand that this task offered a great opportunity to determine a more accurate figure of the Earth.

In order to carry out this task Bessel had to build a chain of triangulation signals to south-western side of the Prussian chain at Trunz - Wildenhof and in north-eastern side of Memel (Klaipeda) – Lepaizi which was part of the Russian chain (Fig.1). The arc to be measured joined Trunz (now Milejevo in Poland) with the lighthouse tower in Memel, altogether 196 kms. The chain of triangles was built at the shore of gulf Frisches Haff, then through Königsberg and Semland peninsula and at last through Kurisches Haff [13].

That Bessel chose as a baseline for these measurements a distance of mere 935 toises or 1823 meters (1 toise= 1.949 m [16]) shows unequivocally that he was a man not troubled by traditions. Up to this time the mantra of geodesists had been that the longer the baseline the better. A baseline of 10 kms was considered normal though to measure it with greatest possible accuracy presented serious problems. According to Bessel: *"Wir zweifelten nicht, durch diese Hülfsmittel eine kürzere Linie mit grosser Genauigkeit bis zu einer beträchtlich längeren vervielfältigen zu können und sahen daher eine grosse Länge der zu messenden Linie für weniger erheblich an, als ihre Nähe bei den Prüfungsapparate der Messstangen und als die Forderung, ihre Länge durch ein gutgeformtes und vielfältige bedingungen zu seiner Prüfung darbietendes Dreieckssystem auf die Dreiecksseite Galtgarben-Condehnen übertragen zu können."* According to Bessel's better judgement for obtaining a certain accuracy by measuring long baseline one may reach the same accuracy by means of measuring a short one if only we measure both the angles and distances extremely carefully with best possible instruments – with "Hülfsmittel"!

The baseline was measured between the manors Trenk and Medniken in five days in August 1834. It began on a small hill, then sank a bit, went over peatlike surface on pastures and grasslands. After having covered 200 toises the baseline reached the fields and more grasslands till it ascended towards the endpoint at Medniken. In the evening of August 11th they reached the point A only 226 toises away – evidently the team was not experienced enough! Next day, having measured the base line from A to Medniken they returned to Trenk and repeated the measurement. Since Bessel was not satisfied with the results, the distance between Trenk to A was measured once more. After painstaking calculations Bessel obtained the result – the base line reduced to the sea level was 934.993124 toises.

Bessel was also very careful in fixing the baseline points. In deep holes (5 feet, 1 Prussian foot = 31.385 cm) with stony foundation the granite boulders were established on top of which the brass cylinders were fixed into the stones. The crosses on these cylinders marked the respective endpoints. Above the ground the points were marked by brickwork pillars, on which a sandstone cube was placed. Again a brass cylinder was placed on that cube while the cylinder extended half an inch from the cube. The cylinder was axially perforated.

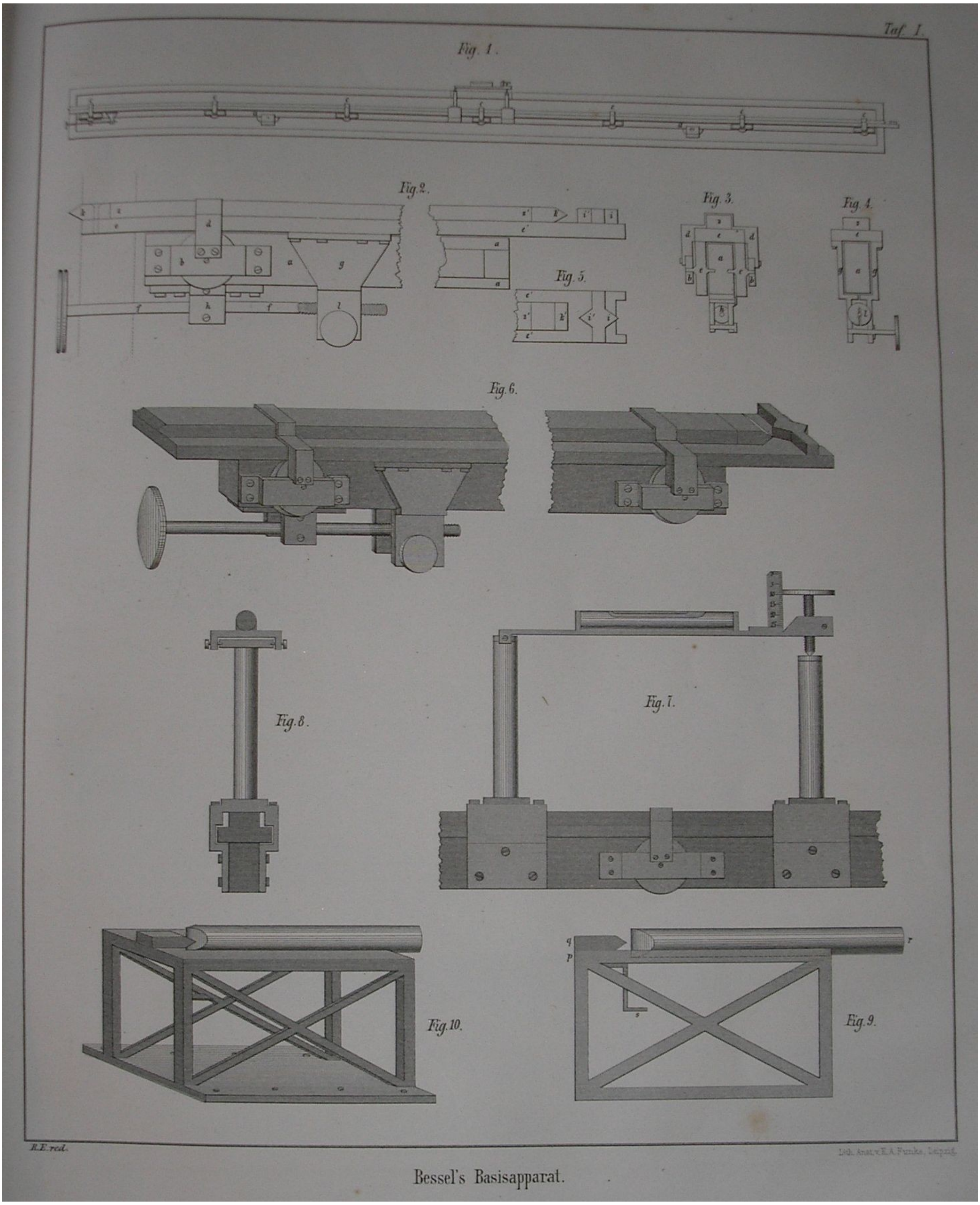
Bessel measured the lengths of the baselines with an instrument of his own construction. The instrument consisted of four rods, each approximately 2 toises long, 12 lignes wide and 3 lignes thick (1 ligne=2.256 mm [18]). The rods were made of two bars - one from iron and the other from zinc, with half the width of that from iron, put one on another and rigidly fixed at the endpoint. The two-metal combination allowed to take the thermal deformation into account without measuring the temperature. The rods were placed in wooden boxes in order to prevent the direct influence of solar rays and humidity on them. The rods were positioned on the baseline with a clearance between them while each rod was supported at seven points on rollers. The clearances were left there deliberately since this effectively excluded accidental displacements of the rods. The length of a clearance was measured with a glass wedge which could be moved out from the rod by means of a screw (the so-called Bessel wedges). The rods were placed on special thick oak platforms in direction of the baseline, and the accuracy of the placement of the rods was checked by levels and passage instruments, i.e. astronomically.

Bessel paid also exclusive attention to scrupulously comparing of the length of his instrument with the standard Toise de Pérou in Paris. It should be mentioned that in 1839 a new standard of length of three Prussian feet was introduced in Prussia which was elaborated by Bessel and which rested upon his theory and experiments with pendulum [14] . During the process of comparison Bessel placed the rods in the water in order to obtain a homogeneous distribution of temperature in them and in that way to reliably estimate the influence of temperature on the rods.

This Bessel instrument was so successful that it was used by geodesists up to the beginning of the 20th century. It should be mentioned that in connection with this work Bessel solved the problem of the least sagging of a rod supported at two points [15].

As far as the arc to be measured was at an angle of 40 degrees of the meridian for some points of the arc both the latitude and longitude had to be measured. Bessel avoided the longitude measurements by measuring the respective azimuths because the longitude measurements were not sufficiently accurate at that time.

For angle measurements Bessel used the transit which was made by Ertel in Munich and designed by Schumacher. The diameter of the horisontal limb was 38 cm and that of the vertical limb – 19 cm. One could read the azimuth with accuracy of 2 arc seconds.



Bessel's Basisapparat.

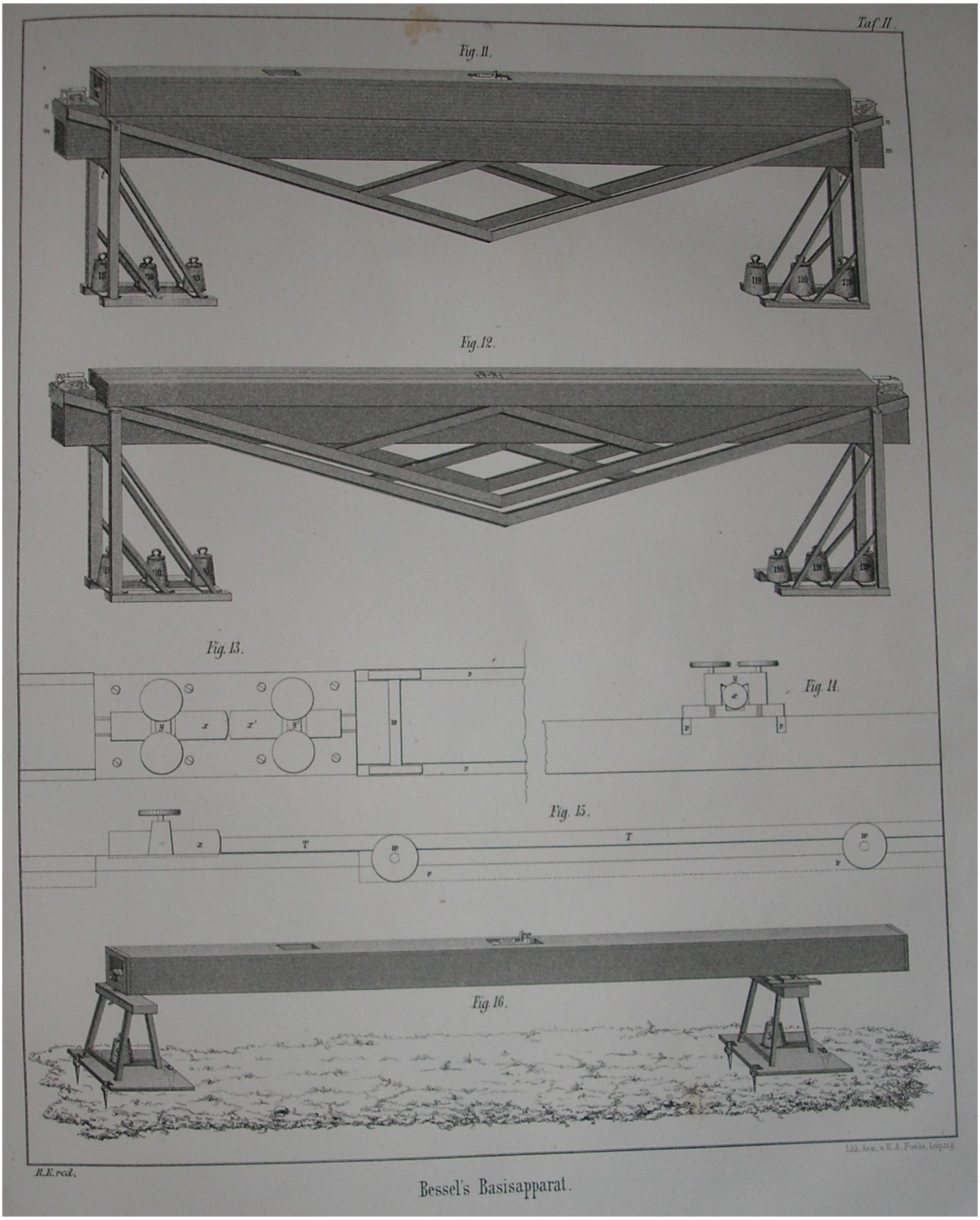
Among Bessel's principal helpers were major Johann Jacob Baeyer from the Prussian General Staff (later General) and lieutenant Kuhlenkamp – perhaps a relative of the owner of a commercial firm in Bremen were Bessel started his independent life.

For sighting the faraway objects Bessel used a polished copper hemisphere with an

approximate diameter of 21 cm, covered with silver and placed on the cylinder protruding from the sandstone cube. During a sunny day a reflected light from that hemisphere could be seen at a distance of 10 km when sighted through a 15 inch theodolite. When it was cloudy a white 2 by 2 feet square with black line of thickness of 10 inches on it was used for sighting. For longer distances – 40 to 50 km - a Gaussian heliotrope was used.
The results of geodetic measurements were to be checked by astronomical observations in order to determine the latitudes and longitudes at some points. This problem was solved by a transportable passage instrument. Here Bessel used his own method which ruled out the instrumental errors for determining the latitudes with this instrument. This method was also used by General C. Tenner in Russia.
The real work for joining the geodetic networks in Prussia and Russia was to begin in 1831. But Bessel and Baeyer could barely start their reconnaisance when epidemic of cholera broke out in Prussia, one of the worst in its history. Bessel and Bayer could start their work again only in 1832.
After the detailed check of the results the two chains were joined and the results were published in a book [13].

4. The Bessel ellipsoid

One of the Bessel's greatest geodetical results was defining a new mathematical model of the Earth's figure. Nowadays we know that as the Bessel ellipsoid. In defining the figure he used his own arc measurement which covered a mere 1° 30' 28”.980. In addition to this he made use of 9 more arc measurements, namely the Peruan (3° 7' 3”.455), the first East-Indian (1° 34' 56”.428), the second East-Indian (15° 15' 40”.728), French (12° 22' 12”.74), English (2° 50' 23”.497), Hannover (2° 0' 57”.42), Danish (1° 31' 53”.306), Russian (8° 2' 28”.907) and Swedish (1° 37' 19”.565). When summing up all the measurements, an arc of almost 50 degrees was covered. The number of astronomical points was 38. Bessel saw his task in finding such an ellipsoid where the distances from its surface to real surface of the Earth were as small as possible. At the same time he planned to orient the axes of the ellipsoid in such a way that all the arc measurements were best presented.
In 1837 Bessel published a paper where he brought out the results for the elements of the Earth's figure: the polar and equatorial radii, the oblateness, analytical values for one degree of meridian and parallel as functions of latitude [9]. He gave also the curvature of



meridian and the distances from the Earth's centre (perhaps it is interesting to point out that the author of his papers in Astronomische Nachrichten was not simply Bessel but at first “Herr Professor und Ritter Bessel” and later “Herr Geh. Rath und Ritter Bessel”!) Unfortunately it happened that four years after the publication of this paper the Paris

Academy of Sciences reported of a substantial error in their calculations – they have used a wrong “Berechnungsart” in elaborating the distance between Montjouy and Mola on Formentera island which resulted in an error of approximately 70 toises (136.5 m). Louis Puissant brought this error before the French Academy and the Academy responded in taking care of repeating the calculations by four independent persons: Largeteau, Daussy, Mathieu and Puissant, and their results coincided in the interval of 5-6 toises. However, Bessel still checked all the French calculations himself and only after he had found that his results coincided very well with the averaged French results, he made all his calculations anew and published the results in 1841 [12]. For the value for the oblateness he obtained 1/299.153.

The Bessel ellipsoid was in use close to 100 years in different countries all over the world. Even the final processing of Struve arc measurements was done on the Bessel ellipsoid [17]. Only after a hundred years enough new data was obtained and the figure of the Earth was more accurately specified and nowadays the accepted result is 1/298.257 ([16], International Earth Rotation and Reference Systems Service, 1989) , so Bessel's error was only 0.3% while Struve's error was 1.2%!

5. Conclusion

Besides of being a brilliant astronomer and mathematician, Bessel was also a very good geodesist. The principles of organising the observations and the theory of both personal and instrumental errors that he had elaborated were all used not only in astronomical but also in geodetic observations. In his works the mathematical apparatus of astronomy and geodesy attained a powerful development. Still we may consider the determination of the exact figure of the Earth one of the Bessel most important results.

Acknowledgements

I am very grateful to Hans-Fredrik Wennström from Lantmäteriet, who introduced me to Maupertuis arc measurements by sending me the Erik Tobé book; to Aili Bernotas from the Tartu University Library who helped me to get the Engelmann book from Freiburg and to Jaan Pelt from Tartu Observatory who helped me with the illustrations.

References

1. Erik Tobé, *Fransysk visit i Tornedalen 1736-1737 (En bok om en gradmätningsexpedition och dess nyckelpersoner)*, Tornedalica, 1986.

2. J.R. Smith, The Struve Geodetic Arc, http://www.fig.net/hsm/struve/struve_arc_smith_2005.pdf

3. J.R. Smith, *The Meridian Arc Measurement in Peru 1735-1745*, FIG XXII International Congress, Washington, D.C USA, April 19-26, 2002.

4. *Abhandlungen von Friedrich Wilhelm Bessel*. Herausgegebenen von Rudolf Engelmann, Dritter Band, Leipzig, Verlag von Wilhelm Engelmann, 1876.

5. F.W. Bessel, *Berechnung eines Dreiecks, dessen Seiten geodätische Linien sind.* Astronomische Nachrichten, vol. 1, p. 85, 1823.

6. F.W. Bessel, *Über die Berechnung der geographischen Längen und Breiten aus geodätischen Vermessungen*. Astronomische Nachrichten, vol. 4, p. 241, 1825.

7. F.W. Bessel, *Bessel's Methode der Berechnung geodätischer Vermessungen.* Astronomische Nachrichten, vol. 6, p. 1, 1828.

8. F.W. Bessel, *Über den Einfluss der Unregelmässigkeiten der Figur der Erde auf geodätische Arbeiten und ihre Vergleichung mit den astronomischen Bestimmungen.* Astronomische Nachrichten, vol. 14, p. 269, 1837.

9. F.W. Bessel, *Bestimmung der Axen des elliptischen Rotationssphäroids.* Astronomische Nachrichten, vol. 14, p. 333, 1837.

10. F.W. Bessel, *Neue Berechnung der Beobachtungen der Polhöhen, auf welchen die zweite in Indien ausgeführte Gradmessung beruhet.* Astronomische Nachrichten, vol. 14, p. 349, 1837.

11. F.W. Bessel, *Über die Polhöhen, welche der Englischen Gradmessung zum Grunde liegen.* Astronomische Nachrichten, vol. 14, p. 381, 1837.

12. F. W. Bessel, *Über einen Fehler in der Berechnung der französischen Gradmessung und seiner Einfluss auf die Bestimmung der Figur der Erde*, Astronomische Nachrichten, Bd. 19, N. 438, S. 97, 1841.

13. *Gradmessung in Ostpreussen und ihre Verbindung mit preussischen und russischen Dreieckskketten.* Ausgeführt von Bessel und Baeyer, in Commission bei F. Dummler, Berlin, 1838.

14. F.W. Bessel, *Untersuchungen über die Länge des einfachen Secundenpendels.* Abhandlungen der Berliner Akademie d. Wiss., Math. Cl., p.1, 1826.

15. F.W. Bessel, *Einfluss der Schwere auf die Figur eines, auf zwei Punkten von gleicher Höhe aufliegenden Stabes.* Darstellung der Untersuchungen und Maassregeln über die Einheit des Preussischen Längenmaasses. Beilage I, Berlin, 1839.

16. http://en.wikipedia.org/wiki/Figure_of_the_Earth.

17. К.К. Лавринович, *Фридрих Вильгельм Бессель*, Москва, “Наука”, 1989, 320 стр.

18. http://www.sizes.com/units/toise.htm

Figure captions

1. Friedrich Wilhelm Bessel – Bessel_portrait.jpg
2. Schemes of measuring rods (1) – measuring_rod.jpg
3. Schemes of measuring rods (2) – measuring_rods_2.jpg
4. Map of the triangulation – scheme_1.jpg

These figures are photographed from the book edited by R. Engelmann [4].