Formulae for study of LID induced diffusion in CP star model atmospheres

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Abstract

The expression for LID acceleration in the inner layers of stellar atmosphere and deeper, where the diffusive approximation for radiative transfer holds, has been given. The equation of continuity and the equation of momentum transfer have been reduced to the form, useful for making computations of evolutionary scenario of isotope segregation based on the model atmosphere.

1. Introduction

About two decades ago Atutov and Shalagin [1], Nasyrov and Shalagin [2] proposed light-induced drift (LID) as effective physical phenomenon for diffusional separation of chemical elements isotopes in the atmospheres of CP stars. Thereafter in former papers [3-6] we studied some general features of the LID phenomenon in the atmospheres of CP stars, assuming that the initial abundance of the studied chemical element and its isotopes is constant throughout the atmosphere and the isotope mixture corresponds to the solar (or terrestrial) one. Presence of stellar wind was ignored. Starting from corresponding initial and boundary conditions and using the computer code SMART [5] composed by us, we have studied the evolutionary abundance changes of mercury and its isotopes due to gravity, radiative pressure gradient and LID. For adequate study of LID effects both the isotopic and the hyperfine splitting of spectral lines are to be taken into account. Due to hyperfine splitting of isotopes with the odd number of nucleons the order of isotope spectral lines will be more mixed, and as a result the picture of evolutionary isotope segregation turns more complicated and slower.

2. Theory

Let us now present some formulae useful for analysis of evolutionary isotope segregation based on the model atmospheres. In the deeper layers of stellar atmospheres, where the diffusive transfer of radiation holds, the radiative transfer turns into the local one and the expression of the monochromatic radiation flux is given by formula

$$F_\nu = FK_R \frac{dB_\nu}{\kappa_\nu dT}, \tag{1}$$

where F is the total radiation flux and the Rosseland opacity integral K_R has the form

$$\frac{1}{K_R} = \int_0^\infty \frac{dB_\nu}{\kappa_\nu dT} d\nu. \tag{2}$$

In the opacity coefficient κ_ν the overlapping spectral lines of the trace element studied play an important role, and thus

$$\kappa_\nu = c_\nu + \sum_j \sigma_j W_j(u_\nu), \tag{3}$$

where c_ν is the continuous opacity coefficient. Summation is made over spectral lines with line profile functions W_j, usually being the Voigt functions, σ_j is the transition cross-section per gram and its argument is given by $u_\nu = (\nu - \nu_j)/\Delta\nu_T$, where $\Delta\nu_T$ is the thermal Doppler width of the spectral line. As we have shown [4], the effective acceleration of LID due to spectral line j can be expressed in the form quite similar to the usual expression of radiative acceleration

$$\alpha_j = \frac{\pi\varsigma_j}{c} \int_0^\infty \frac{\partial W_j(u_\nu)}{\partial u_\nu} F_\nu d\nu \,, \qquad \varsigma_j = q\epsilon\sigma_j, \tag{4}$$

where $q = Mv_T c/2h\nu$ and the efficiency of LID is given by $\epsilon = (C_u - C_l)/(A_u + C_u)$, where C_u and C_l are the collision rates of particles in upper and lower states, respectively, and A_u is the probability (frequency) of spontaneous transitions.
Now we can write the LID acceleration a_j due to spectral line j in the form

$$\alpha_j = \frac{\pi\varsigma_j F K_R}{c} \int_0^\infty \frac{\partial W_j(u_\nu)}{\partial u_\nu} \frac{dB_\nu}{\kappa_\nu dT} d\nu \,. \tag{5}$$

This formula enables to compute formation of isotope segregation due to LID throughout the whole stellar envelope of quiescent stars. For adequate computations the isotopic and hyperfine splitting are to be determined for all ions of all ionization stages. In absolutely quiescent stars there occurs finally drastic segregation of chemical elements. The phenomena which can reduce or cancel the diffusion are stellar wind and turbulence. The needed formulae are easily modified for weak stellar wind, but the needed modification of equations for including turbulence is rather obscure.
The equation of continuity for any isotope i in the plane-parallel stellar atmosphere has the form

$$\frac{d\rho_i}{dt} + \frac{d(\rho_i V_i)}{dr} = 0 \,. \tag{6}$$

The model atmosphere data correspond to standard points, being equidistant on the logarithmic scale of mean optical depth. These points are enumerated as layers n growing downwards. Treating them as standard point values of a continuous parameter n, we can find the values of the derivative dr/dn needed, namely

$$\frac{d}{dr} = -\frac{dn}{dr}\frac{d}{dn} = -\frac{d/dn}{dr/dn} \,. \tag{7}$$

Using the total column density μ, we can write

$$\frac{dr}{dn} = -\frac{d\mu}{\rho dn}. \tag{8}$$

Consequently

$$\frac{d}{dr} = -\gamma\frac{d}{dn}, \qquad \gamma = \frac{\rho}{d\mu/dn} = \frac{\rho}{\mu d\ln\mu/dn} \tag{9}$$

and we can write the equation of continuity in the form

$$\frac{d\rho_i}{dt} = \gamma\frac{d(\rho_i V_i)}{dn} \,. \tag{10}$$

Defining current concentration relative to its initial value ρ_i^0 as C_i by

$$\rho_i = \rho_i^0 C_i \tag{11}$$

and using it, the equation of continuity reduces to

$$\frac{d\ln C_i}{dt} = \frac{\gamma}{\rho_i}\frac{d(\rho_i V_i)}{dn} \,, \tag{12}$$

which avoids possible negative values of C_i in time integration. In addition, for the velocity V_i in the presence of both the diffusion and the wind, we have approximately

$$\rho_i V_i = \rho_i(a_i - g)t - \Delta\frac{d\rho_i}{dr} \,, \qquad \frac{d\rho_i}{dr} = -\gamma\frac{d\rho_i}{dn}, \tag{13}$$

where a_i is the sum of radiative and LID accelerations, t is the mean free time of the particles flight and m is the mean mass of buffer particles. For the diffusion coefficient of trace particles holds $\Delta = kTt/m$. Thus we get

$$\frac{V_i}{\Delta\gamma} = \frac{m(a_i - g)}{kT\gamma} + \frac{d\ln(\rho_i^0 C_i)}{dn}. \tag{14}$$

Now we take into account that for model stellar atmospheres approximately holds

$$\frac{mg}{kT\gamma} = \frac{d\ln\rho_i^0}{dn}. \tag{15}$$

Consequently, for C_i we obtain

$$\frac{d\ln C_i}{dn} = -\frac{ma_i}{kT\gamma} + \frac{V_i}{\Delta\gamma}, \tag{16}$$

which is to be used in the equation of continuity, reducing it to a generalized Fokker-Planck equation. From this equation we obtain as crude predictor

$$\ln C_i = \int_i^N (\frac{ma_i}{kT\gamma} - \frac{V_i}{\Delta\gamma})dn. \tag{17}$$

For the final equilibrium state in the case of no stellar wind $V_i = 0$ and in the case of constant mass loss rate $\rho_i V_i = \dot{m}_i$.

3. Discussion

Our evolutionary model computations show that the values of acceleration are changing essentially slower than the particle densities. Taking into account that the curve-of-growth is implicitly in the Voigt spectral line profile functions, it is clear that weak spectral lines give constant acceleration, thereafter appears the plateau, where acceleration tends to value inversely proportional to ρ_i and in the Lorentz wing region the acceleration is proportional to $\sqrt{\rho_i}$. This circumstance can help to construct approximate expressions for them in the course of evolutionary scenario. This point of view holds for ordinary radiative acceleration. However, for LID the situation is different and essentially more complicated, because the largest contribution is given by asymmetry in the steepest line slopes, generated by overlap of systematically shifted isotope spectral lines. Assuming that during the subsequent evolution accelerations a_i change slowly, we can use the equation (17) for crude prediction of the final concentrations.
During the evolution the time-dependance appears in the mass loss rate, via a_i and also in the values of isotope column densities. Main general formulae for computing diffusion segregation of the trace (impurity) particles in atmospheres of CP stars are Eqs. (12) and (16). As we see, for evolutionary computations we need to find derivatives $\frac{d\ln\rho}{dn}$, $\frac{d\mu}{dn}$ and $\frac{d\gamma}{dn}$, which correspond to buffer gases and several derivatives for each isotope. The derivatives have been found using the 4th order Lagrange interpolation formula for equidistant nodes.
We hope that the present formulae used for the time integration make the physical evolutionary picture visible better, and it can help to estimate the current evolutionary stage of CP stars.
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References

[1] Atutov, S.N., Shalagin, A.M. 1988, Soviet Ast. Lett., 14, 284
[2] Nasyrov, K. A., Shalagin, A. M. 1993, A&A, 268, 201
[3] Sapar A., Aret A. 1995, A&ATr, 7, 1
[4] Aret A., Sapar A. 2002, Astr. Nachr., 323, 1, 21
[5] Sapar A., Poolamäe R. 2003, ASP Conference Series, 288, 95
[6] Sapar A., Aret A., Poolamäe R. 2005, EAS Publ. Series, 17, 341