SCALAR-TENSOR GRAVITY/COSMOLOGY

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I SCALAR-TENSOR THEORY OF GRAVITY

- Standard model of cosmology
- Some motivation to STT and general relativity limit
- Jordan frame vs. Einstein frame
- Observational constraints

II EXAMPLE FROM RANDALL-SUNDRUM I

- RS I, gradient expansion and holographic brane gravity
- ”Master” equation and phase space portraits
- Einstein frame view and problems

III f(R) THEORIES OF GRAVITY

- Generalization to f(R)
- Equivalence between f(R) and scalar-tensor gravity
- Examples, solar system constraints etc

Standard model of physics AD 2000

$$I_{SM} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g}\, \left(\mathcal{L}_{gravity} + \mathcal{L}_\phi + \mathcal{L}_{SM}\right)$$

$$\mathcal{L}_{gravity} = R - 2\Lambda\,, \qquad \mathcal{L}_\phi = -\frac{1}{2}\nabla_\mu\phi\,\nabla^\mu\phi - V(\phi)$$

$\mathcal{L}_{SM} = -\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \frac{1}{2} i g_s^2 (\bar{q}^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu + \bar{G}^a \partial^2 G^a + g_s f^{abc} \partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H - \frac{1}{2} m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \frac{1}{2c_w^2} M \phi^0 \phi^0 - \beta_h [\frac{2M^2}{g^2} + \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - Z^0_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)] - \frac{1}{2} g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac{1}{2} g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w [A_\mu Z^0_\nu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - 2 A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] - \frac{1}{8} g^2 \alpha_h [H^4 + (\phi^0)^4 +$

$$4(\phi^+\phi^-)^2+4(\phi^0)^2\phi^+\phi^-+4H^2\phi^+\phi^-+2(\phi^0)^2H^2]-gMW^+_\mu W^-_\mu H-\frac{1}{2}g\frac{M}{c_w^2}Z^0_\mu Z^0_\mu H-$$

$$\frac{1}{2}ig[W^+_\mu(\phi^0\partial_\mu\phi^- -\phi^-\partial_\mu\phi^0)-W^-_\mu(\phi^0\partial_\mu\phi^+ -\phi^+\partial_\mu\phi^0)]+\frac{1}{2}g[W^+_\mu(H\partial_\mu\phi^- -\phi^-\partial_\mu H)-$$

$$W^-_\mu(H\partial_\mu\phi^+ -\phi^+\partial_\mu H)]+\frac{1}{2}g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0-\phi^0\partial_\mu H)-ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- -W^-_\mu\phi^+)+$$

$$igs_wMA_\mu(W^+_\mu\phi^- -W^-_\mu\phi^+)-ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- -\phi^-\partial_\mu\phi^+)+igs_wA_\mu(\phi^+\partial_\mu\phi^- -$$

$$\phi^-\partial_\mu\phi^+)-\frac{1}{4}g^2W^+_\mu W^-_\mu[H^2+(\phi^0)^2+2\phi^+\phi^-]-\frac{1}{4}g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2+(\phi^0)^2+2(2s_w^2-$$

$$1)^2\phi^+\phi^-]-\frac{1}{2}g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- +W^-_\mu\phi^+)-\frac{1}{2}ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- -W^-_\mu\phi^+)+$$

$$\frac{1}{2}g^2s_wA_\mu\phi^0(W^+_\mu\phi^- +W^-_\mu\phi^+)+\frac{1}{2}ig^2s_wA_\mu H(W^+_\mu\phi^- -W^-_\mu\phi^+)-g^2\frac{s_w}{c_w}(2c_w^2-$$

$$1)Z^0_\mu A_\mu\phi^+\phi^- -g^1s_w^2A_\mu A_\mu\phi^+\phi^- -\bar{e}^\lambda(\gamma\partial+m_e^\lambda)e^\lambda-\bar{\nu}^\lambda\gamma\partial\nu^\lambda-\bar{u}_j^\lambda(\gamma\partial+m_u^\lambda)u_j^\lambda-$$

$$\bar{d}_j^\lambda(\gamma\partial+m_d^\lambda)d_j^\lambda+igs_wA_\mu[-(\bar{e}^\lambda\gamma^\mu e^\lambda)+\frac{2}{3}(\bar{u}_j^\lambda\gamma^\mu u_j^\lambda)-\frac{1}{3}(\bar{d}_j^\lambda\gamma^\mu d_j^\lambda)]+\frac{ig}{4c_w}Z^0_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+$$

$$\gamma^5)\nu^\lambda)+(\bar{e}^\lambda\gamma^\mu(4s_w^2-1-\gamma^5)e^\lambda)+(\bar{u}_j^\lambda\gamma^\mu(\frac{4}{3}s_w^2-1-\gamma^5)u_j^\lambda)+(\bar{d}_j^\lambda\gamma^\mu(1-\frac{8}{3}s_w^2-$$

$$\gamma^5)d_j^\lambda)]+\frac{ig}{2\sqrt{2}}W^+_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)e^\lambda)+(\bar{u}_j^\lambda\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d_j^\kappa)]+\frac{ig}{2\sqrt{2}}W^-_\mu[(\bar{e}^\lambda\gamma^\mu(1+$$

$$\gamma^5)\nu^\lambda)+(\bar{d}_j^\kappa C^\dagger_{\lambda\kappa}\gamma^\mu(1+\gamma^5)u_j^\lambda)]+\frac{ig}{2\sqrt{2}}\frac{m_e^\lambda}{M}[-\phi^+(\bar{\nu}^\lambda(1-\gamma^5)e^\lambda)+\phi^-(\bar{e}^\lambda(1+\gamma^5)\nu^\lambda)]-$$

$$\frac{g}{2}\frac{m_e^\lambda}{M}[H(\bar{e}^\lambda e^\lambda)+i\phi^0(\bar{e}^\lambda\gamma^5e^\lambda)]+\frac{ig}{2M\sqrt{2}}\phi^+[-m_d^\kappa(\bar{u}_j^\lambda C_{\lambda\kappa}(1-\gamma^5)d_j^\kappa)+m_u^\lambda(\bar{u}_j^\lambda C_{\lambda\kappa}(1+$$

$$\gamma^5)d_j^\kappa]+\frac{ig}{2M\sqrt{2}}\phi^-[m_d^\lambda(\bar{d}_j^\lambda C^\dagger_{\lambda\kappa}(1+\gamma^5)u_j^\kappa)-m_u^\kappa(\bar{d}_j^\lambda C^\dagger_{\lambda\kappa}(1-\gamma^5)u_j^\kappa]$$

$$-\frac{g}{2}\frac{m_u^\lambda}{M}H(\bar{u}_j^\lambda u_j^\lambda) - \frac{g}{2}\frac{m_d^\lambda}{M}H(\bar{d}_j^\lambda d_j^\lambda) + \frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar{u}_j^\lambda\gamma^5 u_j^\lambda) - \frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar{d}_j^\lambda\gamma^5 d_j^\lambda) + \bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \frac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + igc_w W_\mu^+(\partial_\mu\bar{X}^0 X^- - \partial_\mu\bar{X}^+ X^0) + igs_w W_\mu^+(\partial_\mu\bar{Y}X^- - \partial_\mu\bar{X}^+ Y) + igc_w W_\mu^-(\partial_\mu\bar{X}^- X^0 - \partial_\mu\bar{X}^0 X^+) + igs_w W_\mu^-(\partial_\mu\bar{X}^- Y - \partial_\mu\bar{Y}X^+) + igc_w Z_\mu^0(\partial_\mu\bar{X}^+ X^+ - \partial_\mu\bar{X}^- X^-) + igs_w A_\mu(\partial_\mu\bar{X}^+ X^+ - \partial_\mu\bar{X}^- X^-) - \frac{1}{2}gM[\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w^2}\bar{X}^0 X^0 H] + \frac{1-2c_w^2}{2c_w}igM[\bar{X}^+ X^0\phi^+ - \bar{X}^- X^0\phi^-] + \frac{1}{2c_w}igM[\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-] + igMs_w[\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-] + \frac{1}{2}igM[\bar{X}^+ X^+\phi^0 - \bar{X}^- X^-\phi^0]$$

The corresponding field equations are:

$$G_{\mu\nu} - g_{\mu\nu}\,\Lambda = \kappa^2\,\left(T_{\mu\nu}^{\phi} + T_{\mu\nu}^{SM}\right)$$

and conservation law for matter:

$$\nabla^\mu\left(T_{\mu\nu}^{\phi} + T_{\mu\nu}^{SM}\right) = 0$$

Standard model of cosmology

$$I_{SM} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,\left(R - 2\Lambda\right) + I_m[g_{\mu\nu},\psi] + I_\phi[g_{\mu\nu},\phi]$$

$$\underset{\text{gravity}}{\uparrow \; (R)} \qquad \underset{\text{matter}}{\uparrow \; (\psi)} \qquad \underset{\text{inflaton}}{\uparrow \; (\phi)}$$

GR + FRW + perfect fluid matter + minimally coupled scalar field ⇒ Standard model

The dynamics of the Universe is governed by Friedmann equations:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}\rho_i + \frac{\Lambda}{3} - \frac{k}{a^2}\,,$$

$$\dot{H} + H^2 \equiv \frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho_i + 3p_i) + \frac{\Lambda}{3}\,,$$

$$\dot{\rho}_i + 3H(\rho_i + p_i) = 0\,.$$

From observations:

$$\Omega_{tot} = \Sigma\,(\rho_i/\rho_{cr}) = \Omega_\Lambda + \Omega_{DM} + \Omega_b \approx 1$$

$$\Omega_\Lambda = 0.73, \qquad \Omega_{DM} = 0.23, \qquad \Omega_b = 0.04$$

Problems with SM and motivation to scalar-tensor gravity(STT)

Main problem in SM:

- *Ad hoc* dark matter(DM), dark energy(DE) and inflation energy (IE)

Motivation to STT:

- Kaluza-Klein compactification of the (4+d)-dimensional gravity $\Longrightarrow$ STT
- Effective low energy string theory in 4-dimensions $\Longrightarrow$ STT
- Effective actions in braneworld models $\Longrightarrow$ STT
- Equivalence of f(R) theories of gravity with STT

Scalar-tensor theory of gravity

$$I_{STT} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-h}\left[\Psi R(h) - \frac{\omega(\Psi)}{\Psi}\Psi^{|\alpha}\Psi_{|\alpha} - 2\kappa^2\, V(\Psi)\right]$$

$$+\int d^4x\sqrt{-h}\mathcal{L}_{matter}$$

- Scalar field is non-minimally coupled to gravity in Jordan frame (scalar and tensor degrees of freedom are mixed)
- Coupling function $\omega(\Psi)$ is characterizing different STT theories
- Gravitational ”constant” is (space)time dependent: $G \sim \Psi^{-1}$
- General relativity limit: $\omega \rightarrow \infty$ and $\omega^{-3}\,\omega' \rightarrow 0$

Conformal transformation from Jordan to Einstein frame

$$g_{\mu\nu} \longrightarrow \bar{g}_{\mu\nu} = \Omega^2(x)\ g_{\mu\nu}\ , \qquad \bar{ds}^2 = \Omega^2(x)\, ds^2$$

leads to canonical action in gravity sector but the conservation law for matter fields is violated

$$\bar{\nabla}_\nu\, \bar{T}^{\mu\nu} = -\frac{1}{\Omega}\,\frac{d\Omega}{d\Psi}\,\bar{T}\,\bar{\nabla}^\mu\Psi \qquad \longleftarrow \text{additional ''force'' in rhs}$$

In order to get canonical action for scalar field we need a redefinition $\Psi \to \phi$

$$d\phi = \pm\sqrt{3/2 + \omega(\Psi)}\,\frac{d\Psi}{\Psi}$$

which is problematic in certain cases.

- Physics ”looks” different in different conformal frames but is it the same?
- What quantities are actually measurable, Jordan frame or Einstein frame?

Parametrized post-Newtonian (PPN) approximation

(Solar system experiments)

Expanding Schwarzschild metric into power series of r_g/r:

$$-g_{00} \approx 1 - \frac{r_g}{r} + \frac{\beta - \gamma}{2} \left(\frac{r_g}{r}\right)^2 \; ; \qquad -g_{rr} \approx 1 + \gamma \, \frac{r_g}{r}$$

For example, Shapiro time delay (radar echo)

$$\Delta t = 2 r_g \left[1 + \frac{1+\gamma}{2} \, ln \left(\frac{4 r_E r_C}{r_0} \right) \right]$$

Eddington parameters γ and β in terms of $\omega(\Psi)$ and $\omega'(\Psi)$:

$$\gamma - 1 \equiv -\frac{1}{\omega_0 + 2} \, , \qquad \beta - 1 \equiv \frac{\kappa^2}{8\pi G} \frac{\omega_0'}{(4 + 2\omega_0)(3 + 2\omega_0)^2}$$

Present observational constraints:

$$\begin{aligned} 4\beta - \gamma - 3 &= (-0.7 \pm 1) \times 10^{-3} \qquad \texttt{LLR} \\ \gamma - 1 &= (2.1 \pm 2.3) \times 10^{-5} \qquad \texttt{Cassini} \end{aligned}$$

Summary of part I:

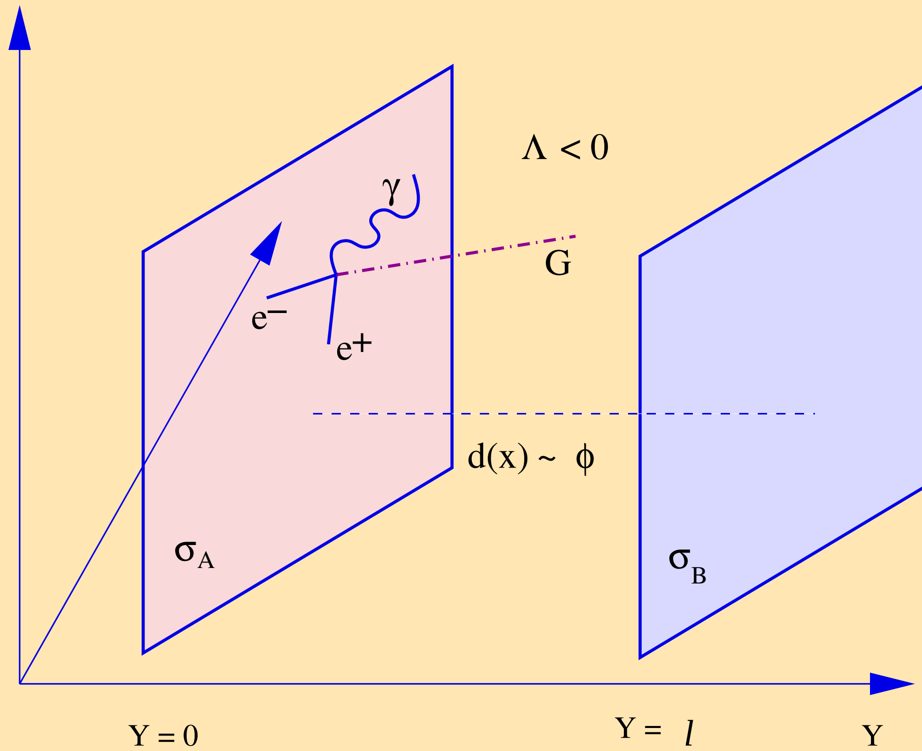
- Scalar-tensor theory is a possible alternative to general relativity which contains scalar degree of freedom in gravitational sector.
  What kind of cosmology STT predicts?

- With the help of conformal transformation and scalar field redefinition it is possible to recover ”canonical” gravitational sector of the theory. But conservation law for matter fields fails.
  Are Jordan and Einstein frame equivalent?

- The observational constraints for scalar-tensor theory are very stringent i.e. at present it should be very close to general relativity.
  Is it true?

Two-brane model

(Randall-Sundrum model I)



- Effective 4-dimensional theory on the brane is in a form of a STT with coupling:

$$\omega(\Psi) = \frac{3}{2}\frac{\Psi}{1-\Psi}\,,$$

- The scalar field Ψ can be interpreted as radion which measures the distance $d(x)$ between the branes

$$d(x) \equiv \int_0^{\ell} \sqrt{g_{55}}\, dy = -\frac{\ell}{2}\ln(1-\Psi)\,, \qquad \Psi \in [0,1]$$

- The second brane matter is in included through the term:

$$+\int d^4x\sqrt{-h}\,(1-\Psi)^2\,\mathcal{L}_m^B$$

- The dynamical equation for H decouples from the scalar field and B-brane matter and its first integral (Friedmann equation with a dark radiation term) reads

$$H^2 = \frac{\kappa^2}{3}\sigma + \frac{\kappa^2}{3}\rho_0\left(\frac{a}{a_0}\right)^{-3\Gamma} - \frac{k}{a^2} + \frac{\kappa^2 C}{3}\left(\frac{a}{a_0}\right)^{-4}$$

The dynamics of the scalar field

(L. Järv, P. Kuusk and M.S.; PRD75:023505, 2007)

A new time variable

$$dN = h_c\, dt, \quad h_c = H + \frac{\dot{\Psi}}{2\Psi}, \quad \frac{df}{dN} = f, \quad p = (\Gamma - 1)\rho$$

in a flat Universe gives a decoupled “master equation” for scalar the field,

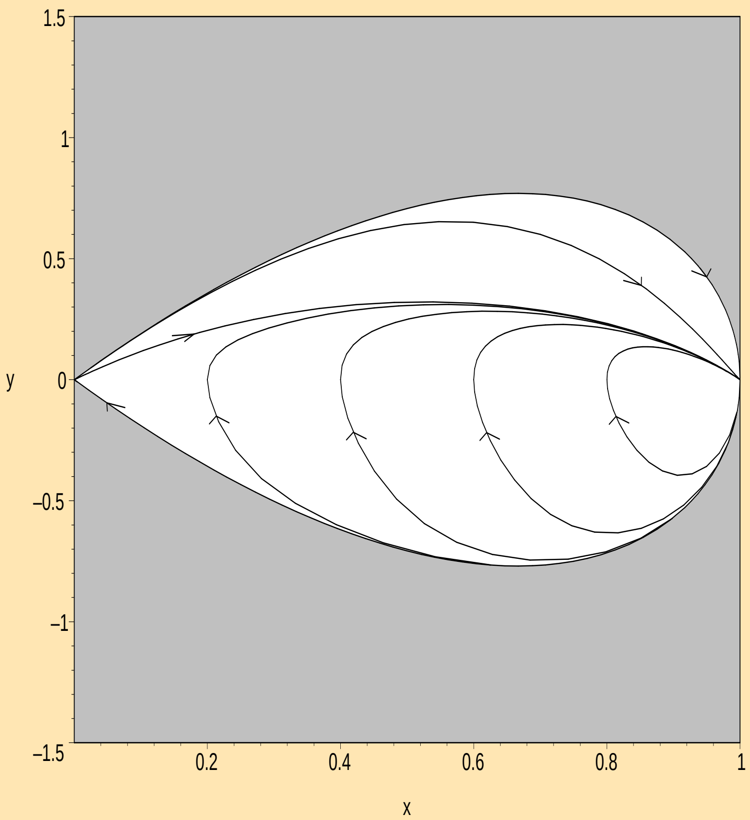
$$8(1-\Psi)\frac{\Psi''}{\Psi} - 3(2-\Gamma)\left(\frac{\Psi'}{\Psi}\right)^3 - 2\left[(4-6\Psi) - (4-3\Gamma)(1-\Psi)W(\Psi)\right]\left(\frac{\Psi'}{\Psi}\right)^2$$
$$+\ \ 12(2-\Gamma)(1-\Psi)\frac{\Psi'}{\Psi} - 8(4-3\Gamma)(1-\Psi)^2 W(\Psi) = 0,$$

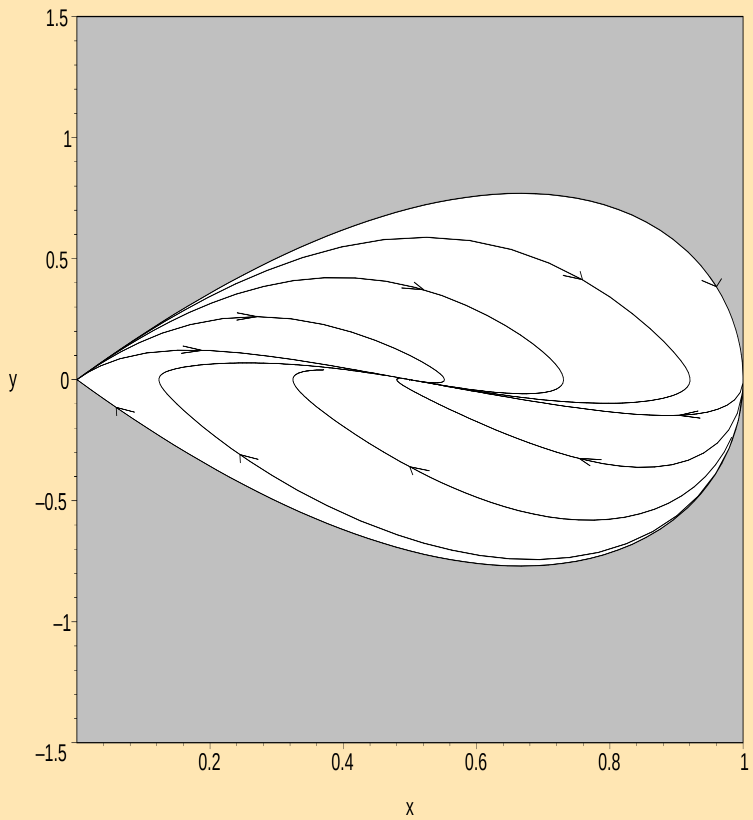
where

$$\Gamma > 0, \qquad W(\Psi) = \frac{1 + (1-\Psi)\,\frac{\rho_0^B}{\rho_0}\left(\frac{1-\Psi}{1-\Psi_0}\right)^{-\frac{3}{2}\Gamma}}{1 + (1-\Psi)^2\,\frac{\rho_0^B}{\rho_0}\left(\frac{1-\Psi}{1-\Psi_0}\right)^{-\frac{3}{2}\Gamma}},$$

$$\Gamma = 0, \qquad W(\Psi) = \frac{1 + (1-\Psi)\,\frac{\sigma^B}{\sigma}}{1 + (1-\Psi)^2\,\frac{\sigma^B}{\sigma}}.$$

Phase portraits for the "master" equation:





Phase portraits ($x = \Psi(p)$, $y = \Psi'(p)$) for the dynamics of scalar field with dust: $\rho_0 = 1$, $\rho_0^B = -0.5$ (left), and $\rho_0 = 0.5$, $\rho_0^B = -1$ (right), $\Psi_0 = 0.5$.

Summary of part II:

- An example of braneworld scenario which leads to a scalar-tensor cosmology with particular coupling function $\omega(\Psi)$ and allows to include the second brane matter.

- Using phase space methods to study the dynamics of the scalar field we saw that general relativity is an attractor for such theory. The exist also an exotic attractor with values $\rho_0^B < 0$, $\rho_0 + (1-\Psi_0)^{3/2}\rho_0^B \geq 0$ which does not correspond to general relativity.

- Recent observations of light element abundances and CMB constrain the dark radiation term to be

$$-0.054 \leq \frac{C}{\rho_0} \leq 0.138$$

- Solar system constraints demand $\Psi \approx 1$, but still do not rule out the model.

Next generalization to f(R):

$$(R - 2\Lambda) \quad \longmapsto \quad f(R) \quad \text{or} \quad f(R, R_{GB})$$

↖ ↗

may be nonlinear

- theory retains the property of general coordinate invariance
- can be seen as the addition of a new scalar degree of freedom, i.e. f(R) $\sim$ STT
- scalar field drives the dark energy (and possibly inflation)

Archetypical example (Carroll, Duvvuri, Trodden, Turner, 2003):

$$f(R) = R - \frac{\mu^4}{R}, \qquad \mu \approx H_0$$

Runaway potential in Einstein frame: $V \sim H_0^2 \, M_{pl} \, e^{-\frac{3}{\sqrt{6}} \frac{\phi}{M_{pl}}}$

$$\Longrightarrow \qquad w = \frac{p}{\rho} = -2/3 \qquad \text{(vacuum case)}$$

Equivalence between the f(R) and STT-s:

(Magnano and Sokolowski 1994, Chiba 2003)

$$I_{JF} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\, f(R) + I_m[g_{\mu\nu},\psi]$$

Introduce a new auxiliary scalar field Q (Lagrange multiplier)

$$I_{JF} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\Big(f'(Q)\,(R-Q) + f(Q)\Big) + I_m[g_{\mu\nu},\psi]$$

If $f''(Q) \neq 0 \quad \Longrightarrow \quad \mathrm{EOM\ of\ Q}: Q = R$.

Redefining $f'(Q) \equiv \Psi$:

$$I_{JF} = \frac{1}{2\kappa^2}\int d^4x\sqrt{g}\,(\Psi R - V(\Psi)) + I_m[g_{\mu\nu},\psi],$$

$$V(\Psi) = [Q(\Psi)\Psi - f(Q(\Psi))]$$

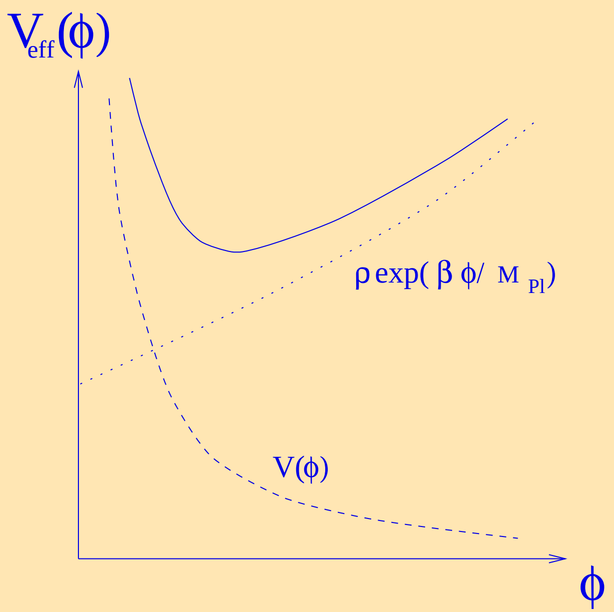
Excluded by SSE!

$\Longrightarrow \omega(\Psi) \equiv 0$ or $\gamma = \frac{1}{2}$ $\swarrow$

f(R) theories that pass solar system tests:

General idea:
we must choose the potential $V(\phi)$ in a way that the solar system tests will be ”deluded”

- Chameleon mechanism: a scalar field lives in effective potential, which depends of the matter density $\Longrightarrow$ mass of the scalar field depends on local matter density.



Effective potential:

$$V_{eff}(\phi) = V(\phi) + \rho_{matter}\,\xi(\phi)$$

Example:

$$f(R) = R - (1-m)\,\mu^{2-2m}\,R^m - 2\Lambda$$

Summary of part III

- The term with negative powers of curvature appear in time-dependent (hyperbolic) compactification of $4+d$-dimensional gravity
  (Norjiri and Odintsov, 2003).

- It is possible to argue that the current acceleration (and inflation in early Universe) arises due to modification of general relativity by additional powers of curvature, i.e. f(R)
  (Carroll, Duvvuri, Trodden, Turner, 2003).

- Chameleon mechanism can pass the solar system tests for some type $f(R)$
  (Khoury and Weltman 2003; Faulkner, Tegmark, Bunn, Mao 2006).

- .....But solar system constraints preclude interesting late time behavior since the acceleration is observationally indistinguishable from a cosmological constant
  (Faulkner, Tegmark, Bunn, Mao 2006/2007).